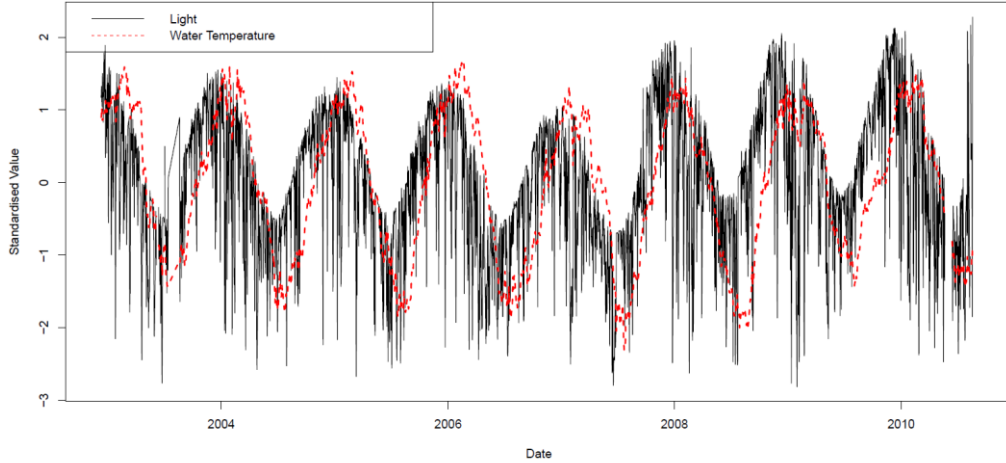


1 Supplementary Material

2 I. Correlation and time lag of water temperature at 6m and light irradiance

3 Figure below shows the correlation between water temperature at 6m and light irradiance at the
4 surface from January 2003 to June 2010 illustrating the clear time lag between the two factors as
5 temperature consistently lags .



6

7 II. Multistate Markov model

8 Assume a finite number of state, $\mathcal{S} = \{1, \dots, R\}$ the transition between the states is governed
9 by a continuous time stochastic process $x(t)$ which has value in \mathcal{S} . Let $H_{t_1}^-$ denote the history
10 of the stochastic process up to time t_1 , the probability of transitioning from state r to state s is
11 $p_{rs}(t_1, t_2) = p(x(t_2) = s | x(t_1) = r, H_{t_1}^-)$. The transitional probability, $p_{rs}(t_1, t_2)$, is governed by
12 the *transition intensity*, q_{rs} which is the instantaneous hazardous rate of moving from state r
13 to state s is

$$q_{rs}(\delta t, H_{t_1}^-) = \lim_{\delta t \rightarrow 0} \frac{p(x(t_1 + \delta t) = s | x(t_1) = r, H_{t_1}^-)}{\delta t}$$

14 The effect of covariates are then included in the model through the intensity matrix, i.e

$$q_{rs}(\delta t, H_{t_1}^-) = q_{rs}^0(t, H_t^-) \exp(\beta_{rs}^T \mathbf{z})$$

15 where $q_{rs}^0(t, H_t^-)$ is the baseline intensity at time t , \mathbf{z} is the covariate vector and β_{rs} is the
16 effect of covariates for transition from state r to s .

17

18 Different assumptions can be made about the dependency between transition rate and time,
19 Meira-Machado et al. (2008) has detailed description of different assumptions and associated
20 models.

21