

# Mathematical models are a powerful method to understand and control the spread of Huanglongbing

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## Supplementary Article 1: Huanglongbing Model Details

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We provide full details of the model here with the equations to outline how the pathogen is transmitted between trees and psyllids. We adapt a non-spatial model of vector-borne disease for malaria (Parham & Michael, 2010). In our model, Equations (S1.1)-(S1.8), citrus trees are categorized as either Susceptible,  $S(t)$  or Infected,  $I(t)$ , and adult psyllids are Susceptible,  $S_V(t)$ ; Exposed,  $E_V(t)$ ; or Infected,  $I_V(t)$ ; where Exposed indicates that the psyllids are infected but are not yet able to pass the disease on to another tree. Once infected, psyllids remain so for their entire lifespan. Successful transmission between psyllid and tree can only occur when psyllids feed off the phloem of the tree; the feeding rate is independent of grove size thus transmission is frequency-dependent. A very small rate of natural death of susceptible trees occurs and we include roguing of infected trees; together these trees are categorized as Removed,  $R(t)$ . However, removed trees are immediately replaced in the grove by susceptible trees, keeping the grove size constant. Thus, the removed category exists to keep track of how many trees have been removed and replaced over time; it does not represent alive trees in the grove. We assume that the grove has 100% susceptible trees initially, with psyllids feeding freely from the trees. At time 0, we introduce one infected tree. We consider the change in numbers of susceptible, infected and removed trees for the following 20 years to understand the effects of the initial infection on the whole grove.

$$\frac{dS}{dt} = -\frac{ab}{N}I_V S + r_1 I \quad (\text{S1.1})$$

$$\frac{dI}{dt} = \frac{ab}{N}I_V(t - \tau)S(t - \tau) - r_1 I \quad (\text{S1.2})$$

$$\frac{dR}{dt} = rS + r_1 I \quad (\text{S1.3})$$

$$\frac{dS_V}{dt} = \lambda - \frac{ac}{N}IS_V - \mu S_V \quad (\text{S1.4})$$

$$\frac{dE_V}{dt} = \frac{ac}{N}IS_V - \frac{ace^{-\mu/\phi}}{N}I(t - EIP)S_V(t - EIP) - \mu E_V \quad (\text{S1.5})$$

$$\frac{dI_V}{dt} = \frac{ace^{-\mu/\phi}}{N}I(t - EIP)S_V(t - EIP) - \mu I_V \quad (\text{S1.6})$$

$$N = S + I \quad (\text{S1.7})$$

$$V = S_V + E_V + I_V. \quad (\text{S1.8})$$

We measure time in years so all rates are per year.  $a$  is the feeding rate of the psyllid on the trees,  $b$  is the probability that a susceptible tree becomes infected from contact with an infected psyllid, and  $c$  is the probability that a susceptible psyllid becomes infected from contact with an infected tree. Hence  $bc$  is the vector competence. We impose a time delay  $\tau$  to represent the length of the exposed period when a tree is infected but not yet infectious. This time delay is long (approximately 6 months (Gottwald, 2010)), hence we use a fixed time delay of length  $\tau$  to represent this more accurately than using a simple exponentially distributed exposure period.  $r$  is the natural death rate of susceptible trees.  $r_1$  is the rate of removal of infectious trees by roguing. We assume all removed and dead trees are replanted

with susceptible trees, hence the  $r_1 I$  term (and the lack of  $rS$  term) in equation (S1.1).  $N$  is the total number of alive trees in the grove and  $V$  is the total number of adult psyllids. Note that for simplicity, we assume that there is no natural death of exposed trees, so all trees that are infected will progress to being infectious.

Adult psyllids have a fixed birth rate  $\lambda$ , which we explain in more detail below. The development of eggs and nymphs is included within this birth rate term. The death rate of psyllids is  $\mu$ .  $EIP$  is the extrinsic incubation period, the length of time until the psyllid becomes infectious.  $\phi$  ( $= 1/EIP$ ) is the parasite development rate, the rate at which the bacteria develops within the psyllid. Again we include the incubation period  $EIP$  as a strict time delay similar to the tree equations. However, many psyllids will die during the exposed period and so we include an exponential term,  $e^{-\mu/\phi}$ , as a discounting term, calculating the expected number of psyllids that will survive the exposed period to become infectious.

The size of the vector population can be modeled as a function of the demographic parameters. The probability of having  $V$  vectors at time  $t$  tends to a Poisson distribution with mean  $\frac{\lambda}{\mu}$  (Parham & Michael, 2010). The birth rate,  $\lambda$ , can be expressed as the product of the number of eggs laid over a lifetime  $\frac{EFD}{\mu}$  (where  $EFD$  is the number of eggs laid per female per year) and the probability the eggs survive to adulthood  $p_{ea}$ , over the average duration of the immature stages (Mordecai *et al.*, 2013). Hence,

$$\lambda = \frac{EFD p_{ea} MDR}{\mu} \quad (\text{S1.9})$$

where  $MDR$  is the vector development rate, i.e.  $1/\text{time}$  for a vector to develop from egg to adult.

Mathematical models of disease systems often use  $R_0$ , the basic reproductive number, as a measure of disease prevalence. It is a measure of how many secondary hosts will become infected if one initial host is infected in a naïve population. The equation for  $R_0$  for Equations (S1.1)-(S1.8) is given by:

$$R_0 = \left( \frac{V a^2 b c}{N r_1 \mu} e^{(-\mu/\phi)} \right)^{1/2} = \left( \frac{EFD p_{EA} MDR a^2 b c}{N r_1 \mu^3} e^{(-\mu/\phi)} \right)^{1/2}. \quad (\text{S1.10})$$

This equation for  $R_0$  can be understood by considering how disease propagates through the system. It is dependent on the ratio of vectors to hosts ( $V/N$ ), how many successful bites there are from host to vector ( $ac$ ) over the average lifespan of an infected host ( $1/r_1$ ), how many bitten vectors survive the exposed period ( $e^{-\mu/\phi}$ ), and how many successful bites those vectors make on hosts ( $ab$ ) over the lifespan of an infected vector ( $1/\mu$ ). One useful aspect of this definition is that if  $R_0 > 1$ , the epidemic will spread throughout the grove, whereas if  $R_0 < 1$ , the disease will die out. This is because on average, if  $R_0 > 1$ , one infected tree infects as least one other tree causing the disease to continue rather than die out.

## Parameter Values

Our model aims to use realistic parameter values for each of the parameters, which necessitates analyzing how each of the parameters varies throughout the year due to environmental conditions. Psyllids are ectotherms and thus will be sensitive to fluctuations in temperature

both daily and throughout the year. We include this in our model by incorporating the data presented in Liu & Tsai (2000), which determines life history parameters at different temperatures. Following the work of Mordecai *et al.* (2013), we expect thermal response curves to be unimodal, and left-skewed, since metabolic reaction rates tend to increase up to an optimal temperature, then decline due to protein degradation and other processes. However, as the data were limited, we allow for linear response curves. We fit thermal response curves for  $EFD$ ,  $p_{ea}$ ,  $MDR$  and  $\mu$  assuming Brière, quadratic or linear functions, and determine the best fitting curve by the Akaike Information Criterion (AIC). For  $\mu$ , we also used a data point from Hall *et al.* (2011) to determine the effect of freezing on longevity of psyllids. The parameter fits are included in Table S1.1 with the data and best fit for the parameters plotted in Figure S1.1.

Parameter	Curve	Curve Parameters
$p_{ea}$	Linear	$a = 0.4719(0.103)$ , $b = 0.0109 (0.004)$
$MDR$	Brière	$c = 5.286 \times 10^{-5}(1.822 \times 10^{-5})$ , $T_0 = 10.02(2.858)$ , $T_m = 34.17(2.335)$
$EFD$	Brière	$c = 0.011(0.004)$ , $T_0 = 13.049(3.231)$ , $T_m = 30.845(0.896)$
$1/\mu$	Quadratic	$q = -0.14221(0.04717)$ , $r = 4.31998(1.59958)$ , $s = 31.25498(15.96673)$

Table S1.1: Thermal response fits for psyllid life history parameters. Brière [ $cT(T - T_0)(T_m - T)^{1/2}$ ], quadratic, [ $qT^2 + rT + s$ ] and linear [ $a + bT$ ] curves were fitted for each parameter, with Akaike Information Criterion (AIC) used to determine the best fit. Best fit values are listed with the standard deviations in parenthesis. See Table S1.2 for psyllid parameter definitions. We fit longevity ( $1/\mu$ ) rather than the death rate ( $\mu$ ) for ease. Note that in this table the parameters are daily rates but for (S1.1)-(S1.8) we change to yearly rates (see Table S1.2).

Note that we plot longevity (i.e., the inverse of the mortality rate;  $1/\mu$ ) in Figure S1.1D. From Figure S1.1A, we see that the best fit for parameter  $p_{ea}$ , the probability of egg to adult survival, is a linear curve, which contradicts our current understanding of unimodal response curves as discussed earlier. Thus, more data collection on this parameter, for a wider range of temperatures, would be useful to understand more clearly how it varies with temperature. However, we do not believe it is of much concern for our current model as Figure 3A (main text; sensitivity analysis) indicates that parameter  $p_{ea}$  is not influential on the model results.

A full list of all parameter values, with references, is provided in Table S1.2. All the parameters in this table are yearly rates, as used in equations (S1.1)-(S1.8).

We include temperature variation in the model over a year with the following equation:

$$T(t) = 25(1 + 0.3 \sin(2\pi t)) \quad (\text{S1.11})$$

which allows temperature to vary between 17.5°C and 32.5°C throughout the year, similar to average temperatures in Florida (Duever *et al.*, 1994).

Including temperature variation in the psyllid life-history traits results in  $R_0$  changing throughout the year dependent on temperature (Figure S1.2).  $R_0$  declines twice each year, both in summer and winter, but the higher temperatures are more deleterious and results in  $R_0$  having a value of 0 for a significant portion of the year. At those high temperatures the psyllids are unable to produce any eggs. No transmission of infection would occur when  $R_0 = 0$  and hence the number of infected trees will decline due to roguing during those times.

Parameter	Interpretation	Value	Reference
$a$	Feeding rate of vector	$0.05 * 365$	A
$b$	Probability of transmission from infected psyllid to susceptible tree	0.1	B
$c$	Probability of transmission from infected tree to susceptible psyllid	0.3	B
$\tau$	Exposed time delay for tree	0.5	C
$r$	Death rate of susceptible tree	$1/60$	C
$r_1$	Removal rate of infected trees	$1/10$	D
$r_2$	Removal rate of treated trees	$1/15$	-
$L$	Longevity (in days)	$-0.14221T^2 + 4.3198T + 31.25498$	E, F
$\mu$	Death rate of psyllids	$1/L * 365$	-
$EIP(1/\phi)$	Parasite incubation period	$10/365$	A
$EFD$	Eggs per female psyllid per year	$0.0107 * 365 * T(T - 13)(30.8 - T)^{1/2}$	E
$p_{ea}$	Probability of egg psyllid surviving to adulthood	$0.47192 + 0.0109T$	E
$MDR$	Development rate of psyllid	$(5.286 \times 10^{-5}) * 365 * T(T - 10.02)(34.17 - T)^{1/2}$	E
$\rho$	Effectiveness of insecticide spray	$0.6 - 0.99$	G
$m$	Number of days spraying insecticide per year	$10 - 60$	H
$\kappa$	Proportion of trees treated with antibiotic	$0.3, 0.9$	-
$\epsilon$	Efficiency of antibiotic treatment	$0.6, 0.83$	J

Table S1.2: Parameter descriptions and values for the model described in Equations (S1.1)-(S1.8).  $T$  refers to temperature, as given by (S1.11). Rates are per year. References are: A - Pelz-Stelinski *et al.* (2010), B - Lee *et al.* (2015), C - Gottwald (2010), D - Martini *et al.* (2015), E - Liu & Tsai (2000), F - Hall *et al.* (2011), G - Rogers (2008), H - Spreen *et al.* (2006), J - Spreen *et al.* (2006).

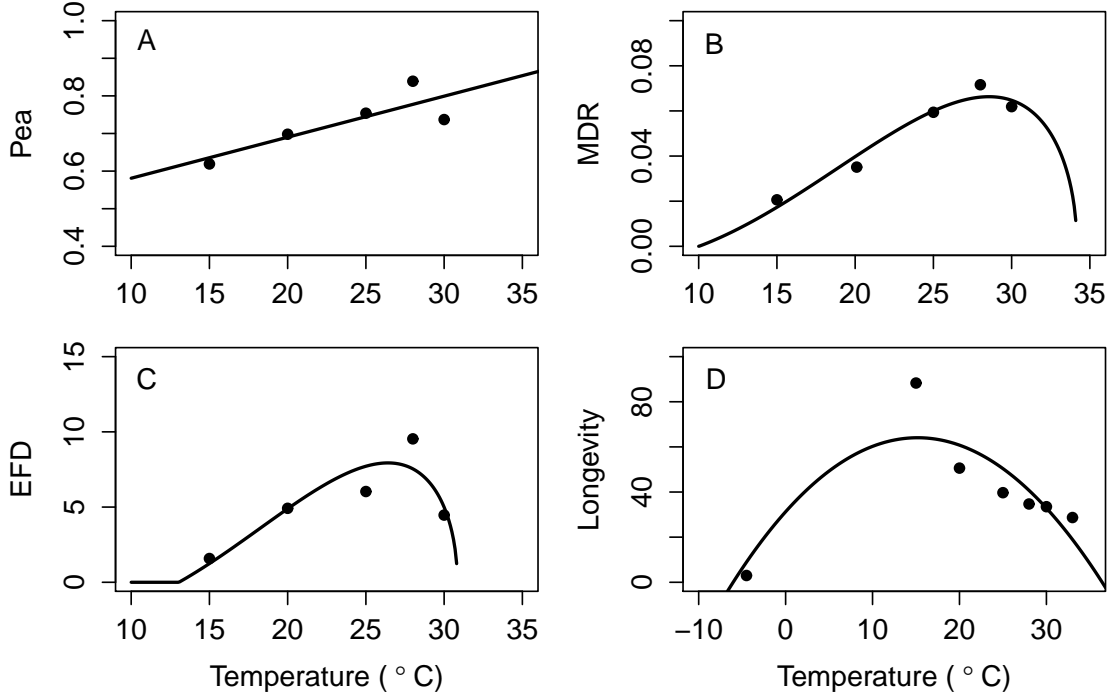


Figure S1.1: Thermal response curves for the psyllid life history parameters, using data from Liu & Tsai (2000). For  $D$ , we also used a data point from Hall *et al.* (2011) to determine the effect of freezing on longevity of psyllids. We fit linear, Brière, and quadratic curves for each parameter with the best fit (determined by AIC) shown here. In A, probability of egg to adult survival ( $p_{ea}$ ); in B, psyllid development rate ( $MDR$ ); in C, psyllid fecundity ( $EFD$ ); in D, psyllid longevity ( $1/\mu$ ). For parameter values of the fits, see Table S1.1. Note the different temperature range for  $D$ , due to the inclusion of a freezing reference point.

## Sensitivity Analysis

We perform sensitivity analysis of  $R_0$  for the temperature dependent parameters by analyzing the impact of these parameters on  $R_0$ . We calculate how each parameter contributes to the temperature sensitivity of  $R_0$ ,  $\frac{dR_0}{dT}$ :

$$\begin{aligned} \frac{dR_0}{dT} &= \frac{\partial R_0}{\partial EFD} \frac{dEFD}{dT} + \frac{\partial R_0}{\partial p_{EA}} \frac{dp_{EA}}{dT} + \frac{\partial R_0}{\partial MDR} \frac{dMDR}{dT} + \frac{\partial R_0}{\partial \mu} \frac{d\mu}{dT} \\ &= \left( \frac{1}{2EFD} \frac{dEFD}{dT} + \frac{1}{2p_{EA}} \frac{dp_{EA}}{dT} + \frac{1}{2MDR} \frac{dMDR}{dT} - \left( \frac{3}{2\mu} + \frac{1}{2\phi} \right) \frac{d\mu}{dT} \right) R_0 \quad (S1.12) \end{aligned}$$

Here  $\frac{\partial R_0}{\partial x}$  is the partial derivative of  $R_0$  with respect to parameter  $x$  and  $\frac{dx}{dT}$  is the derivative of parameter  $x$  with respect to temperature, using the equations in Table S1.2.

We use equation (S1.12) to analyze the relative contribution of each parameter to overall temperature sensitivity of  $R_0$ , as shown in Figure 3 in the main text. In the figure, we plot the full equation to indicate  $R_0$ 's overall temperature sensitivity (black line) while each colored line is only the contribution for that parameter - i.e. the term in equation (S1.12) corresponding to that parameter.

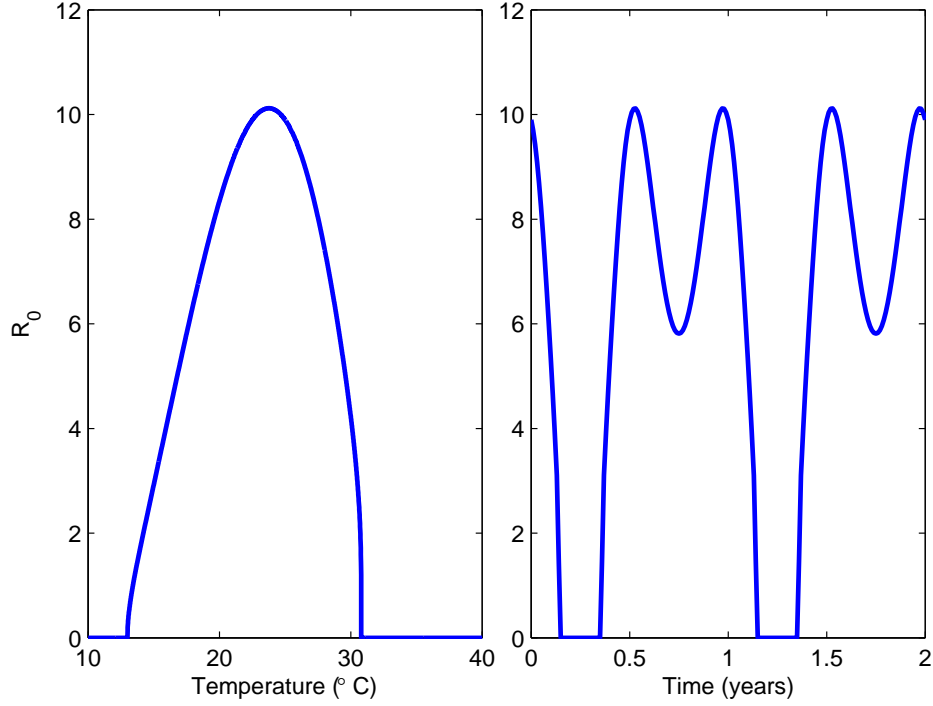


Figure S1.2:  $R_0$  is plotted against temperature ( $^{\circ}\text{C}$ ) and time (years) to show the seasonal fluctuations. The year begins in spring.  $R_0$  is given by (S1.10) with parameter values as in Table S1.2 and temperature varying yearly according to (S1.11).

## Insecticide Intervention

Insecticide spraying is currently the main method of control in widespread use in Florida (Qureshi *et al.*, 2014). The efficiencies of different insecticides vary, as well as the average duration of reduction of psyllids. The average insecticide efficiency is between 90-100% but some sprays can have an average efficiency as low as 53% (Qureshi *et al.*, 2014). When insecticide is applied to the groves it targets all adult psyllids through increasing their death rate. It will also reduce the birth rate of psyllids, to represent the insecticide killing eggs and nymphs. We analyse the effectiveness of the strategy using two aspects of the system: the peak number of trees infected and the total profit made. We include costs of the interventions to provide an aspect of cost-effectiveness as yields and profits are of primary interest to the citrus growers.

In our model, spraying occurs twice in a year, in spring and autumn. Both spring and autumn sprays will involve spraying for the same number of consecutive days. But between simulations we vary the total number of days spraying each year. In order to include insecticide spraying of  $m$  days in our model we use the following term:

$$ins(t) = \begin{cases} 365\rho & \text{if } t \in [t_n, t_n + \frac{m}{2*365}] \text{ or } t \in [t_n + \frac{1}{2}, t_n + \frac{m}{2*365} + \frac{1}{2}] \\ 0 & \text{else} \end{cases} \quad (\text{S1.13})$$

where  $ins(t)$  denotes the level of insecticide occurring at time  $t$ ,  $t_n$  is an integer between 0 and 20 to represent years we solve for,  $\rho$  is the effectiveness of the spray (between 0 and 1)

per day and  $m$  is the number of days in the year the insecticide is sprayed. Therefore,  $\frac{m}{2}$  is the number of days in each spraying session. We multiply  $\rho$  by 365 to switch to a yearly rather than daily rate (to be consistent with (S1.1)-(S1.8)).

Insecticide spraying affects psyllids by killing them thus we include this term as an additional death rate. As we have a fixed birth rate independent of the number of psyllids, we need to take into account that insecticide will reduce the number of psyllids and hence this fixed rate should also decline. Thus, we multiply the birth rate of psyllids by this term. This leads to the following equations for insecticide intervention:

$$\frac{dS}{dt} = -\frac{ab}{N}I_V S + r_1 I \quad (\text{S1.14})$$

$$\frac{dI}{dt} = \frac{ab}{N}I_V(t - \tau)S(t - \tau) - r_1 I \quad (\text{S1.15})$$

$$\frac{dR}{dt} = rS + r_1 I \quad (\text{S1.16})$$

$$\frac{dS_V}{dt} = \lambda ins(t) - \frac{ac}{N}IS_V - \mu S_V - ins(t)S_V \quad (\text{S1.17})$$

$$\frac{dE_V}{dt} = \frac{ac}{N}IS_V - \frac{ace^{-\mu/\phi}}{N}I(t - EIP)S_V(t - EIP) - \mu E_V - ins(t)E_V \quad (\text{S1.18})$$

$$\frac{dI_V}{dt} = \frac{ace^{-\mu/\phi}}{N}I(t - EIP)S_V(t - EIP) - \mu I_V - ins(t)I_V \quad (\text{S1.19})$$

$$N = S + I \quad (\text{S1.20})$$

$$V = S_V + E_V + I_V. \quad (\text{S1.21})$$

In the main text, we vary both  $\rho$  and  $m$ , the effectiveness of spraying and the total number of days spraying per year to understand the efficiency of insecticide at reducing disease prevalence.

## Costs

The costs and profits of the insecticide intervention strategy is an important aspect in making informed decisions about the strategy to adopt. We focus on the key costs and profits in this early model to give a good indication of cost-effectiveness of the strategy (Table S1.3). The costs we include are: cost of removing a tree and planting a new tree (if the tree has died naturally or been removed due to infection) and cost of 1 day of insecticide spraying. We also include some basic profits, namely, the profits from susceptible, infected and treated trees over a year. The profits made from infected trees will change as their infection status changes; we use an average value as we do not keep track of how long individual trees are infected for.

The cost of one day of insecticide spraying depends on the effectiveness of the insecticide, such that more effective insecticides cost more money to use. We use the following equation



to determine cost:

$$\text{Insecticide cost per day} = \frac{15.075\rho}{1.203 - \rho}. \quad (\text{S1.22})$$

This leads to a cost which varies from \$15 for  $\rho = 0.6$  to \$70 for  $\rho = 0.99$  where  $\rho$  is once again insecticide effectiveness.

One notable cost which we do not include is that of checking trees for infection. This can actually be rather costly depending on the number of days a year which are spent checking for infection and the method used for detection. However, there is such variation in how often people check trees, and the cost itself is difficult to calculate and will change between groves, that for this model we exclude it.

We include a discount factor by multiplying future costs and profits by the term  $e^{-rt}$ . This allows for more realistic results over a 20 year time period. Cost and profit values used in the model are included in Table S1.3. We have used profits and costs from Stansly *et al.* (2014) to assess likely values.

Parameter	Value
Discount factor	0.001
Cost to replace tree	\$35
Cost of 1 day of insecticide spraying	Eqn. (S1.22)
Profit from a susceptible tree per year	\$388
Proportion of profit for infected tree	40%

Table S1.3: Cost and profit values used in our model. The discount factor is based upon a current inflation rate of 0.1%. The cost and profit values are estimated from Spreen *et al.* (2006); Stansly *et al.* (2014).

We incorporate the income calculations into our model by adding an additional differential equation:

$$\frac{dP}{dt} = 388S + (0.4)388I \quad (\text{S1.23})$$

where  $P(t)$  is the income at time  $t$ . We can then calculate costs and total profits using the solutions to the delay differential equations (S1.1)-(S1.8) and (S1.23), and the parameters in Table S1.3.

## Methods

We numerically solve the delay differential equations (S1.1)-(S1.8) using Matlab, solver `dde23`. The grove has 101 susceptible trees and 500 susceptible psyllids. At time 0, one of these 101 susceptible trees becomes infected. That is, we are assuming that psyllids are already present but infection is not, before we start the simulation. This is in comparison to a case where psyllids invade an entirely susceptible grove, but some of the psyllids may already be infectious. Our scenario represents Florida well, since psyllids existed there for 7 years before HLB arrived, therefore it is likely that most groves had psyllids present before they became infected.

We analyze the dynamics of the infection over the following 20 years. For the intervention model, we apply the control strategy from time 0 and consider different control parameters

whilst keeping the disease parameters the same as their values in Table S1.2. In no cases did applying the intervention strategy lead to complete removal of the disease from the grove.

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