Algorithm 1 Graph Split

Input: Semantic Subgraph, \( Q \)

1: if \( |V(Q)| \leq 4 \) then
2: \( D = \{ Q \} \)
3: else
4: \( Q \) is converted to an undirected graph
5: if cliqueCheck \( (Q) \) then
6: cliqueSplit \( (Q) \)
7: else
8: \( ON = v_{\text{max}}(Q); v_1; v_2; max = -1 \)
9: for \( i \in V(\forall j \in V \land i \neq j) \) do
10: if shortestPath \( (i, j) > max \) then
11: \( max = \text{shortestPath} \ (i, j); v_1 = i; v_2 = j \)
12: end if
13: end for
14: \( D_1 = \) all nodes in \( \delta(v_1, ON) \)
15: \( D_2 = \) all nodes in \( \delta(v_2, ON) \)
16: for \( v \in V \) do
17: if \( v \notin (V(D_1) \cup V(D_2)) \) then
18: allocate left over nodes
19: end if
20: end for
21: \( D = \{ D_1, D_2 \} \)
22: while \( |(V)|d \in D > 4 \) do
23: return Graph Splitting\( (d) \)
24: end while
25: end if
26: end if

Output: \( D \)

Graph Split

In Graph Splitting the shortest path is calculated using the implementation of Dijkstra's shortest path algorithm (Dijkstra, 1959) provided by the JGraphT library. If a semantic subgraph is found to be a clique then an \( ON \) is picked at random and \( D \) is made up of a random partitioning of \( Q \), such that \( |V(D_1)| \) and \( |V(D_2)| \) are equal if \( n \) is odd or \( |V(D_1)| = |V(D_2)| + 1 \) if \( n \) is even (\( ON \) is in both graphs). If \( Q \) is not a clique then semantic subgraph membership of remaining nodes depends upon the edges in which they participate. Given semantic subgraphs \( D_1 \) and \( D_2 \), node subgraph membership is allocated using the following rules:

1. If a node shares an edge with only one of \( D_1 \) or \( D_2 \) it will be allocated to that subgraph.
2. If a node shares an edge with nodes from both \( D_1 \) and \( D_2 \) it will be allocated to the subgraph with which it shares the greatest number of edges.
3. If a node shares an equal number of edges with nodes in \( D_1 \) and \( D_2 \) it will be allocated to the subgraph containing the fewest nodes.
4. If a node shares an equal number of edges with nodes in \( D_1 \) and \( D_2 \) and \( D_1 \) and \( D_2 \) have an equal number of nodes it will be allocated \( D_1 \).

Edges \( (e \in E(Q)) \) are then allocated to \( D_1 \) if both nodes of \( e, v_1, v_j \), are \( v \in V(D_1) \) or to \( D_2 \) if both nodes of \( e, v_i, v_j \), are \( v \in V(D_2) \). When \( v_i, v_j \) are not present in the same graph then \( e \) is not included in the split graphs. Instead \( e \) will be checked for during the mapping stage of the algorithm. If either of the subsequent graphs still have a nodeset greater than 3 after an initial split, the algorithm may be called again and the resulting subgraphs are searched for and results mapped systematically.

In Fig. 1 (below), it is clear that searching using a purely topological approach (Normal\_0.0) is infeasible for a subgraph with a \( |V(Q)| \) of 7 when the target graph reaches a nodeset size of \( 2 \times 10^3 \). Performance is improved when searching using a semantic threshold of 0.8 (Normal\_0.8). However, search time is still costly when the target graph reaches a nodeset size of \( 3 \times 10^4 \). When Graph Splitting is used prior to a semantic search with a semantic threshold of 0.8 (Split), a search for semantic subgraphs with a \( |V(Q)| \) of 7 can be completed on a large target graph (\( 1 \times 10^5 \)) in a reasonable, finite time.

These results show that Graph Splitting allows us to search for subgraphs that were previously intractable using an exhaustive exact approach; both topologically and, to a lesser extent, semantically.

The Venn Diagrams (Fig. 2 below) demonstrate that the same results are obtained (i.e. the same set of matches are returned) when using Graph Splitting or a non-split approach.
Figure 1: **Graph Splitting search time in comparison to a non-split approach.** *Note:* Semantic subgraphs were created at random with a $|V(Q)|$ of 7. A purely topological search was then completed without Graph Splitting (Normal 0.0). Semantic searches were then carried out without Graph Splitting (Normal 0.8), and finally, with Graph Splitting (Split). Searches were carried out on random target graphs where $|V(G)|$ was between $1 \times 10^3$ and $1 \times 10^5$. 
Figure 2: **Graph Splitting matches in comparison to matches returned using a non-split approach.**  
*Note:* Semantic subgraphs were created at random with a $|V(Q)|$ of 6 and 7, respectively. Semantic subgraphs were then searched for using either Graph Splitting (Split_search) or a non-split approach (Normal_Search) using the algorithm with one parameter; every element of the match needed to pass the ST (left). Returned sets of matches were compared and the percentage of matches between the two sets calculated. All tests were run using the algorithm with two alternate parameters i) where every element of the match needed to pass the ST (left), and ii) all elements had to cumulatively pass the ST (right).